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Plastic flow coupled with a crack in some one- and two-dimensional quasicrystals

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Abstract

An analysis of a group of slip dislocations coupled with a crack dislocation group in some one- and two-dimensional quasicrystals is given. The extent of the plastic zone and the amount of the dislocation slip have been determined in a very explicit form.

1. Introduction

Plastic deformation of quasicrystals is a new subject. Among others, the group of Urban, Messerschmidt and co-workers studied the problem through a series of experimental observations [1–7]. They found that the mechanism of plastic deformation in such a case consists of dislocation motion. Trebin and co-workers [8] carried out a theoretical study on the dislocation motion for a decagonal quasicrystal. They also performed some molecular dynamics simulations to discuss the interaction between a crack and dislocations in the material [9]. Fan [10], and Fan *et al* [11] obtained a solution for stationary and moving dislocations of one-dimensional hexagonal quasicrystals. Li *et al* [12, 13] have constructed analytic solutions for dislocations in two-dimensional decagonal quasicrystals of point groups $10mm$ and $10, \overline{10}$ respectively. They [10, 14, 15] also found the analytic solution for a Griffith crack in a one-dimensional hexagonal, and two-dimensional decagonal and octagonal quasicrystal. These can also be referred to in monograph [10] or review paper [16]. Recently Liu and Fan [17] developed a complex variable function method for solving the elasticity problems of an elliptic notch in a $10mm$ point group two-dimensional decagonal quasicrystal. These solutions can provide some information for discussing the interaction between dislocations and cracks in quasicrystals. In this paper, the classical BCS model for crystals [18, 19] is extended to quasicrystals. We focus on the common character of dislocation solutions in one- and two-dimensional quasicrystals, on the basis of which the integral equation method can uniformly manipulate the interaction between cracks and dislocations, and by

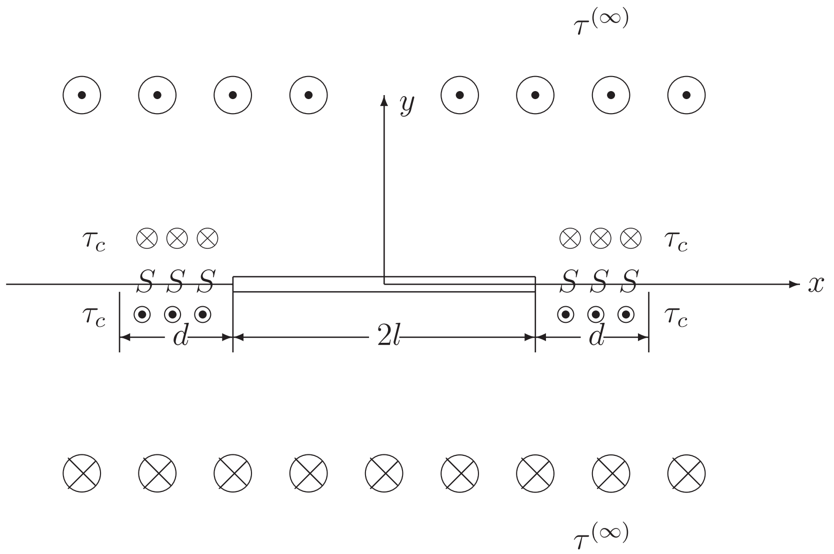

Figure 1. Schematic picture of a screw dislocation pile up group coupled with a crack in a one-dimensional hexagonal quasicrystal.

which analytic solutions are obtained. Though we can use the complex variable method to solve the problems, this is not given here since the derivation would tend to be protracted.

2. A survey on the solutions of dislocations in quasicrystals

A quasicrystal is a structure of solids involving two different low energy elementary excitations—phonon and phason, where the former is in the parallel space (physical space) while the latter is in a vertical space (complementary space). The existence of the phason field causes an essential difference in the elasticity of quasicrystals compared to that of crystals.

Consider a quasicrystal in a rectilinear system (x, y, z). If the one-dimensional quasicrystal along the z axis atomic arrangement is quasiperiodic, and that along the x–y plane atomic arrangement is periodic, then we have phonon displacements u_x, u_y, u_z and phason displacement w_z (and $w_x = w_y = 0$). The Burgers vector of a straight dislocation along the axis z for the quasicrystal is $\boldsymbol{b} = (b_1^{\parallel}, b_2^{\parallel}, b_3^{\parallel}, 0, 0, b_3^{\perp})$. (A comprehensive discussion on the introduction to the six-dimensional Burgers vector can be found in [20].) Since the components $b_1^{\parallel}, b_2^{\parallel}$ for the case are pure phonon Burgers vector components, we will not discuss them here. We are interested in only the components $b_3^{\parallel}$ and $b_3^{\perp}$, which are in phonon–phason coupling. The stress field induced by $b_3^{\parallel}$ and $b_3^{\perp}$ of the dislocation located at the origin of the coordinates (figure 1) in a one-dimensional hexagonal quasicrystal [10] is

$$\begin{aligned}
\sigma_{xz} = \sigma_{zx} &= -\frac{C_{44} b_3^{\parallel}}{2\pi}\left(\frac{y}{r^2}\right) - \frac{R_3 b_3^{\perp}}{2\pi}\left(\frac{y}{r^2}\right) \\
\sigma_{yz} = \sigma_{zy} &= -\frac{C_{44} b_3^{\parallel}}{2\pi}\left(\frac{x}{r^2}\right) - \frac{R_3 b_3^{\perp}}{2\pi}\left(\frac{x}{r^2}\right) \\
H_{zx} &= -\frac{K_2 b_3^{\perp}}{2\pi}\left(\frac{y}{r^2}\right) - \frac{R_3 b_3^{\parallel}}{2\pi}\left(\frac{y}{r^2}\right) \\
H_{zy} &= -\frac{K_2 b_3^{\perp}}{2\pi}\left(\frac{x}{r^2}\right) + \frac{R_3 b_3^{\parallel}}{2\pi}\left(\frac{x}{r^2}\right)
\end{aligned} \tag{1}$$

where $r = \sqrt{x^2 + y^2}$, σ_{ij} denotes the phonon stress components, H_{ij} the phason components, C_{44} the phonon elastic constant, K_2 the phason elastic constant, and R_3 the phonon–phason coupling elastic constant.

Secondly we will consider a two-dimensional quasicrystal in the same coordinate system. Assume that the atomic arrangement along z is periodic, while in the x–y plane the atomic arrangement is quasiperiodic. We have phonon displacements u_x, u_y, u_z and phason ones w_x, w_y (and $w_z = 0$). If there is a straight dislocation along the axis z in the quasicrystal, then one has Burgers vector $\boldsymbol{b} = (b_1^{\parallel}, b_2^{\parallel}, b_3^{\parallel}, b_1^{\perp}, b_2^{\perp}, 0)$. For a point group $10mm$ two-dimensional quasicrystal, Li and Fan [12] obtained the solution in terms of the Fourier transform; the results are the same as those given by Ding *et al* [20] by the Green function method. We will list here only the relevant stresses induced by components $b_1^{\parallel}$ and $b_1^{\perp}$ [12]:

$$
\begin{aligned}
\sigma_{yx} = \sigma_{xy} &= \frac{b_1^{\parallel}}{\pi}\frac{(L+M)(MK_1-R^2)}{(L+2M)K_1-R^2}\frac{x(x^2-y^2)}{r^4} \\
&\quad + \frac{b_1^{\perp}}{\pi}\frac{(L+M)(K_1-K_2)R^2}{[(L+2M)K_1-R^2]R}\frac{2xy^2(y^2-x^2)}{r^6} \\
\sigma_{yy} &= \frac{b_1^{\parallel}}{\pi}\frac{(L+M)(MK_1-R^2)}{(L+2M)K_1-R^2}\frac{y(x^2-y^2)}{r^4} - \frac{b_1^{\perp}}{\pi}\frac{(L+M)(K_1-K_2)R^2}{[(L+2M)K_1-R^2]R}\frac{y^3(3x^2-y^2)}{r^6} \\
H_{yx} &= -\frac{b_1^{\parallel}}{\pi}\frac{(L+M)(MK_1-R^2)(K_1-K_2)R}{[(L+2M)K_1-R^2](MK_1-R^2)}\frac{x^3(3y^2-x^2)}{r^6} \\
&\quad - \frac{b_1^{\perp}}{4\pi}\frac{(L+M)(K_1-K_2)R^2}{[(L+2M)K_1-R^2](MK_1-R^2)}x \\
&\quad \times\left[\frac{2(x^2-y^2)}{r^4} + \frac{(x^2-y^2)(3x^2-y^2)(3y^2-x^2)}{r^8}\right] \qquad (2) \\
H_{xy} &= -\frac{b_1^{\parallel}}{\pi}\frac{(L+M)(K_1-K_2)R}{(L+2M)K_1-R^2}\frac{xy^2(3x^2-y^2)}{r^6} + \frac{b_1^{\perp}}{4\pi}\frac{(L+M)(K_1-K_2)R^2}{[(L+2M)K_1-R^2](MK_1-R^2)} \\
&\quad \times\left[\alpha\frac{x^2}{r^2} + \frac{2xy^2(3x^2-y^2)(3y^2-x^2)}{r^8}\right] \\
H_{yy} &= -\frac{b_1^{\parallel}}{\pi}\frac{(L+M)(K_1-K_2)R}{(L+2M)K_1-R^2}\frac{xy^2(3x^2-y^2)}{r^6} \\
&\quad - \frac{b_1^{\perp}}{4\pi}\frac{(L+M)(K_1-K_2)R^2}{[(L+2M)K_1-R^2](MK_1-R^2)}y \\
&\quad \times\left[\frac{2(x^2-y^2)}{r^4} + \frac{(x^2-y^2)(3x^2-y^2)(3y^2-x^2)}{r^8}\right].
\end{aligned}
$$

Here σ_{xx} and H_{xx} are omitted, whereas

$$
r = \sqrt{x^2+y^2}, \qquad L = C_{12}, \qquad M = (C_{11}-C_{12})/2 = C_{66},
$$

$$
\alpha = \frac{[(L+2M)K_1-R^2](MK_1-R^2)}{(L+M)(K_1-K_2)R^2}\left[2 + \frac{(L+M)K_2-R^2}{(L+2M)K_1-R^2} + \frac{MK_2-R^2}{MK_1-R^2}\right].
$$

For a point group $10, \overline{10}$ two-dimensional quasicrystal, we offer the solution for a dislocation by the Fourier transform method [13]. The solution is not found by other methods (including the Green function method). Here we will list only the stress field induced by the components $b_1^{\parallel}, b_1^{\perp}$ of the Burgers vector for the dislocation:

$$
\begin{aligned}
\sigma_{yx} = \sigma_{xy} &= \frac{b_1^{\|}}{\pi}\frac{(L+M)(MK_1-R^2)}{(L+2M)K_1-R^2}\frac{x(x^2-y^2)}{r^4} \\
&\quad -\frac{b_1^{\perp}}{\pi}\frac{(MK_1-R^2)(L+2M)(K_1-K_2)R}{[(L+2M)K_1-R^2](MK_1-R^2)} \\
&\quad \times\left[\frac{R_1}{R}\frac{xy^2(3x^2-y^2)}{r^6}+\frac{R_2}{R}\frac{x^2(3y^2-x^2)}{r^6}\right] \\
\sigma_{yy} &= \frac{b_1^{\|}}{\pi}\frac{(L+M)(MK_1-R^2)R}{(L+2M)K_1-R^2}\frac{y(x^2-y^2)}{r^4}-\frac{b_1^{\perp}}{\pi}\frac{(MK_1-R^2)(L+M)(K_1-K_2)R}{[(L+2M)K_1-R^2](MK_1-R^2)} \\
&\quad \times\left[\frac{R_1}{R}\frac{y^3(3x^2-y^2)}{r^6}+\frac{R_2}{R}\frac{xy^2(3y^2-x^2)}{r^6}\right] \\
H_{yx} &= -\frac{b_1^{\|}}{\pi}\frac{(L+M)(K_1-K_2)R}{[(L+2M)K_1-R^2]}\left[-\frac{R_1}{R}\frac{x^3(3y^2-x^2)}{r^6}+\frac{R_2}{R}\frac{x^3(3x^2-y^2)}{r^6}\right] \\
&\quad -\frac{b_1^{\perp}}{4\pi}\frac{(L+M)(K_1-K_2)R^2}{[(L+2M)K_1-R^2](MK_1-R^2)} \\
&\quad \times\left\{\frac{R_1^2-R^2}{R^2}\left[\frac{2(x^2-y^2)}{r^4}+\frac{(x^2-y^2)(3x^2-y^2)(3y^2-x^2)}{r^8}\right]\right. \\
&\quad \left.\times\frac{4R_1R_2}{R_2}\frac{xy(3x^2-y^2)(3y^2-x^2)}{r^8}\right\} \qquad (3) \\
H_{xy} &= -\frac{b_1^{\|}}{\pi}\frac{(L+M)(K_1-K_2)R}{(L+2M)K_1-R^2}\left[\frac{R_1}{R}\frac{xy^2(3x^2-y^2)}{r^6}+\frac{R_2}{R}\frac{x^2y(3y^2-x^2)}{r^6}\right] \\
&\quad +\frac{b_1^{\perp}}{4\pi}\frac{(L+M)(K_1-K_2)R^2}{[(L+2M)K_1-R^2](MK_1-R^2)} \\
&\quad \times\left\{\beta\frac{x}{r^2}+y\left[\frac{R_1^2-R_2^2}{R^2}\frac{2xy(3x^2-y^2)(3y^2-x^2)}{r^6}\right.\right. \\
&\quad \left.\left.+\frac{2R_1R_2}{R^2}\left(\frac{2(x^2-y^2)}{r^4}+\frac{(x^2-y^2)(3x^2-y^2)(3y^2-x^2)}{r^8}\right)\right]\right\} \\
H_{yy} &= -\frac{b_1^{\|}}{\pi}\frac{(L+M)(K_1-K_2)R}{(L+2M)K_1-R^2}\left[\frac{R_1}{R}\frac{x^2y(3y^2-x^2)}{r^6}-\frac{R_2}{R}\frac{xy^2(3x^2-y^2)}{r^6}\right] \\
&\quad -\frac{b_1^{\perp}}{4\pi}\frac{(L+M)(K_1-K_2)^2R^2y}{[(L+2M)K_1-R^2](MK_1-R^2)}\left\{\frac{R_1^2-R_2^2}{R^2}\right. \\
&\quad \times\left[\frac{2(x^2-y^2)(3y^2-x^2)}{r^4}+\frac{(x^2-y^2)(3x^2-y^2)(3y^2-x^2)}{r^8}\right] \\
&\quad \left.+\frac{4R_1R_2}{R^2}\frac{xy(3x^2-y^2)(3y^2-x^2)}{r^8}\right\}.
\end{aligned}
$$

Here σ_{xx} and H_{xx} are not listed, whereas

$$R=\sqrt{R_1^2+R_2^2},$$

$$\beta=\frac{[(L+2M)K_1-R^2](MK_1-R^2)}{(L+M)(K_1-K_2)R^2}\left[2+\frac{(L+2M)K_2-R^2}{(L+M)K_1-R^2}+\frac{MK_2-R^2}{MK_1-R^2}\right].$$

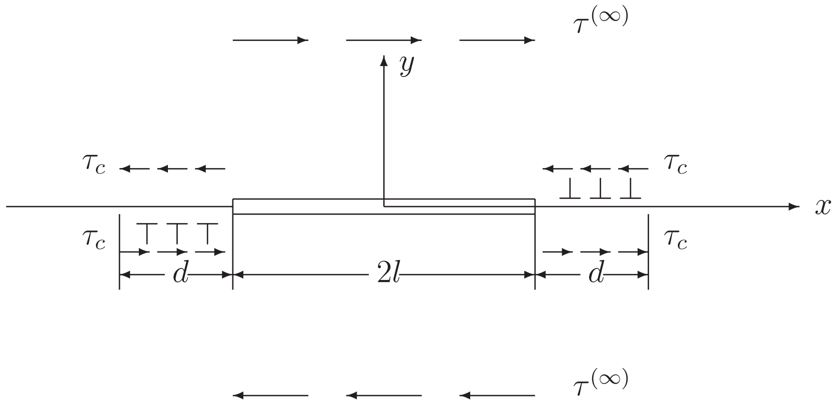

Figure 2. Schematic picture of an 'edge' dislocation pile up group coupled with a crack in a two-dimensional decagonal quasicrystal.

The difference between the solutions for point group $10mm$ and point group $10, \overline{10}$ lies in the phonon–phason coupling elastic constants: the former has only one coupling elastic constant R while the latter has two different coupling elastic constants R_1 and R_2. This makes the computation more difficult for the solution of the problem for point group $10, \overline{10}$.

Solutions corresponding to other components of the Burgers vectors have not been listed, again in consideration of limitation in space.

Solutions of a dislocation in a point group $8mm$ two-dimensional quasicrystal were given in [10, 21], involving more elastic constants. However, the results are too protracted to be given at length. The solution on a dislocation in point group $12mm$ two-dimensional quasicrystal is simpler where the phonon and phason fields are decoupled. It too will not be listed here.

The most important shear stresses in the solutions listed above present a common feature at $y = 0$:

$$\sigma_{ij}(x, 0) = A\frac{1}{x} \tag{4}$$

where A is a constant different for different quasicrystal systems and which is made up of phonon, phason and phonon–phason coupling elastic constants and as well as the corresponding Burgers components reflecting the influence of quasicrystal systems and sample configurations.

3. Plastic flow around crack tips

Assume that there is a Griffith crack with length $2l$ along the z axis in a one- or two-dimensional quasicrystal, subjected to a stress at infinity $\sigma_{yz} = \tau^{(\infty)}$ for the one-dimensional quasicrystal (figure 1) or $\sigma_{yx} = \tau^{(\infty)}$ for the two-dimensional quasicrystal (figure 2), and that around the crack tip there is a dislocation pile up of length d (figure 1 or 2). The material is subjected to a plastic flow stress τ_c which is a material constant, and the size d is unknown so far.

The interaction between a crack and dislocation pile up in a quasicrystal is a problem of plasticity theory of the material, which is a nonlinear problem mathematically and physically due to the irreversible deformation. But we have assumed already that the stress distribution within the plastic zone is known, i.e.,

$$y = 0, \quad l < |x| < l + d: \qquad \sigma_{yz} = \tau_c \qquad \text{for a one-dimensional quasicrystal,} \tag{5}$$

and

$$y = 0, \quad l < |x| < l + d: \qquad \sigma_{yx} = \tau_{\mathrm{c}} \qquad \text{for a two-dimensional quasicrystal,} \tag{6}$$

respectively, so that the nonlinear problem is linearized mathematically, i.e., the plasticity problem is reduced to an 'equivalent' elasticity problem. This makes the mathematical treatment extremely simplified. It is well-known that the superposition principle can be used for linear problems. Using the superposition principle, the boundary value problem for the interaction between a crack and a screw dislocation pile up group in a one-dimensional quasicrystal can be reduced to

$$\begin{array}{ll} \sqrt{x^2+y^2} \to \infty: & \sigma_{ij} = 0, \quad H_{ij} = 0 \\ y = 0, \quad |x| < l: & \sigma_{yz} = -\tau^{(\infty)}, \quad H_{yz} = 0 \\ y = 0, \quad l < |x| < l + d: & \sigma_{yz} = -\tau^{(\infty)} + \tau_{\mathrm{c}}, \quad H_{yz} = 0 \end{array} \tag{7}$$

and for the interaction between a crack and a straight 'edge' dislocation pile up group of a two-dimensional quasicrystal, the boundary value problem is reduced to

$$\begin{array}{ll} \sqrt{x^2+y^2} \to \infty: & \sigma_{ij} = 0, \quad H_{ij} = 0 \\ y = 0, \quad |x| < l: & \sigma_{yx} = -\tau^{(\infty)}, \quad H_{yx} = 0, \quad \sigma_{yy} = 0, \quad H_{yy} = 0 \\ y = 0, \quad l < |x| < l + d: & \sigma_{yx} = -\tau^{(\infty)} + \tau_{\mathrm{c}}, \quad H_{yx} = 0, \quad \sigma_{yy} = 0, \quad H_{yy} = 0. \end{array} \tag{8}$$

Though the governing equations of the two problems above are quite different (i.e., the governing equation for one-dimensional quasicrystals is a fourth order partial differential equation while for two-dimensional quasicrystals it is an eighth order partial differential equation; see appendix A), and the mathematical structure of the boundary conditions related to crack–dislocations interaction presents some common features, they can uniformly be reduced to an integral equation by introducing the dislocation density $f(\xi)$ such as

$$\int_L \frac{f(\xi)\,\mathrm{d}\xi}{\xi - x} = \frac{\tau(x)}{A} \tag{9}$$

where ξ is the coordinate of the source point and x the coordinate of the field point at the real axis, L represents the real axis, and

$$\tau(x) = \begin{cases} -\tau^{(\infty)}, & |x| < l \\ -\tau^{(\infty)} + \tau_{\mathrm{c}}, & l < |x| < l + d \end{cases} \tag{10}$$

$$A = \begin{cases} -\dfrac{C_{44} b_3^{\parallel}}{2\pi} + \dfrac{R_3 b_3^{\perp}}{2\pi} & \text{for case (a)} \\ \dfrac{b_1^{\parallel}}{\pi} \dfrac{(L+M)(MK_1 - R^2)}{(L+M)K_1 + (MK_1 - R^2)} & \text{for case (b)} \\ \dfrac{b_1^{\parallel}}{\pi} \dfrac{(L+M)(MK_1 - R^2)}{(L+2M)K_1 - R^2} & \\ \quad + \dfrac{b_1^{\perp}}{\pi} \dfrac{(MK_1 - R^2)(L+2M)(K_1 - K_2)R_2}{[(L+2M)K_1 - R^2](MK_1 - R^2)} & \text{for case (c).} \end{cases} \tag{11}$$

In the above expressions we distinguish the following cases, namely:

(a) for a screw dislocation pile up in a one-dimensional hexagonal quasicrystal;
(b) for a straight 'edge' dislocation pile up in a point group $10mm$ two-dimensional quasicrystal; and
(c) for a straight 'edge' dislocation pile up in a point group 10, $\overline{10}$ two-dimensional quasicrystal.

Equation (9) is a Cauchy type singular integral equation. By the theory of Muskhelishvili [22] and (10) we obtain the solution of (9):

$$f(x) = -\frac{1}{\pi^2 A}\sqrt{\frac{x+(l+d)}{x-(l+d)}}\int_L \sqrt{\frac{\xi-(l+d)}{\xi+(l+d)}}\tau(\xi)\,\frac{\mathrm{d}\xi}{\xi - x}$$

$$= -\frac{1}{\pi^2}\sqrt{\frac{x+(l+d)}{x-(l+d)}}\left\{\mathrm{i}\left[2\tau_c\cos^{-1}\left(\frac{l}{l+d}\right)-\tau^{(\infty)}\pi\right]\right\}$$

$$+\frac{1}{\pi^2}\left[\cosh^{-1}\left|\frac{(l+d)^2-lx}{(l+d)(l-x)}\right| - \cosh^{-1}\left|\frac{(l+d)^2+lx}{(l+d)(l+x)}\right|\right]. \quad (12)$$

To avoid lengthy derivations, the details of the calculation have to be omitted here. Because the dislocation density $f(x)$ should be a real number, the factor multiplying the imaginary number i must be zero; this leads to

$$2\tau_c\cos^{-1}\left(\frac{l}{l+d}\right)-\tau^{(\infty)}\pi = 0 \quad (13)$$

or

$$d = l\left[\sec\left(\frac{\pi\tau^{(\infty)}}{2\tau_c}\right)-1\right]. \quad (13')$$

Hence the extent of the dislocation pile up is determined. Further explanation for the physical meaning of the calculation can be found in appendix B.

From solution (12) we can evaluate the amount of dislocations $N(x)$, i.e.,

$$N(x) = \int_0^x f(\xi)\,\mathrm{d}\xi. \quad (14)$$

Substituting (12) (coupled with (13)) into (14), we can get $N(l+d)$ and $N(l)$, so the amount of the dislocation motion is

$$\delta = b_3^{\|}[N(l+d)-N(l)] = \frac{2b_3^{\|}l}{\pi^2 A}\left(\ln\frac{l+d}{l}\right) = \frac{2b_3^{\|}l}{\pi^2 A}\left(\ln\sec\left(\frac{\pi\tau^{(\infty)}}{2\tau_c}\right)\right)$$
$$A = -\frac{1}{2\pi}(C_{44}b_3^{\|} - R_3 b_3^{\perp}) \quad (15)$$

for the interaction between a crack and a screw dislocation pile up group in a one-dimensional quasicrystal, and

$$\delta = b_1^{\|}[N(l+d)-N(l)] = \frac{2b_1^{\|}l}{\pi^2 A}\left(\ln\frac{l+d}{l}\right) = \frac{2b_1^{\|}l}{\pi^2 A}\left(\ln\sec\left(\frac{\pi\tau^{(\infty)}}{2\tau_c}\right)\right) \quad (16)$$

$$A = \begin{cases} \dfrac{b_1}{\pi}\dfrac{(L+M)(MK_1-R^2)}{(L+M)K_1+(MK_1-R^2)} & \text{for point group } 10mm \\ \dfrac{b_1^{\|}}{\pi}\dfrac{(L+M)(MK_1-R^2)}{(L+2M)K_1-R^2} & \\ \quad + \dfrac{b_1^{\perp}}{\pi}\dfrac{(MK_1-R^2)(L+2M)(K_1-K_2)R_2}{[(L+2M)K_1-R^2](MK_1-R^2)} & \text{for point group } 10, \overline{10} \end{cases} \quad (17)$$

for the interaction between a crack and a straight 'edge' dislocation pile up group in a two-dimensional decagonal quasicrystal.

If $\tau^{(\infty)}/\tau_c \ll 1$, and by making a Taylor expansion to $\sec(\pi\tau^{(\infty)}/2\tau_c)$ and holding to the second term, then (13) gives

$$\frac{d}{l} \simeq \frac{\pi^2}{8}\left(\frac{\tau^{(\infty)}}{\tau_c^2}\right)^2 \tag{18}$$

so $d/l \ll 1$ too in this case, which is the small scale plastic deformation case. From (B.1) we know that

$$K = \sqrt{\pi(l+d)}\tau^{(\infty)} \simeq \sqrt{\pi l}\tau^{(\infty)} \tag{19}$$

in which the subscript of K is omitted for simplicity; then (18) can be expressed by

$$d \simeq \frac{\pi^2}{8}\left(\frac{K}{\tau_c}\right)^2. \tag{20}$$

In a similar manner (15), (16) have the approximate forms in this case, respectively

$$\delta \simeq \frac{b_3^{\|}}{2\pi A}\frac{K^2}{\tau_c} \tag{21}$$

for the interaction between an anti-plane crack and a screw dislocation pile up in one-dimensional quasicrystals, where A is given by the second formula of equation (15), and

$$\delta \simeq \frac{b_1^{\|}}{2\pi A}\frac{K^2}{\tau_c} \tag{22}$$

for the interaction between a plane crack and an ‘edge’ dislocation pile up in two-dimensional quasicrystals, where A is defined by (17).

From (20) to (22) it can be seen that the extent of plastic flow and amount of dislocation slip can be expressed by the corresponding stress intensity factors, which are the parameters of linear elastic fracture theory, in the small scale plastic deformation case. But (20)–(22) do not hold apart from in the case of small scale plastic deformation. This means that we must do a complete plastic analysis for medium and large scale deformation cases, and (13), (15) and (16) are necessary.

The discussion here is limited by the background of present knowledge of solutions of cracks and dislocations of quasicrystals. So far the results in this respect are very limited due to the tremendous complexity of the material and the huge difficulty of the mathematical treatment. Even if this is so, the above analysis reveals that:

(1) the plastic zone is coplanar with the crack surface;
(2) the plastic zone is a dislocation pile up with counter direction;
(3) the amount of slip around the crack tip is equal to $b_3^{\|}[N(l+d)-N(l)]$ for a screw dislocation group of one-dimensional quasicrystals, and $b_1^{\|}[N(l+d)-N(l)]$ for an ‘edge’ dislocation group of two-dimensional quasicrystals.

Of course the theoretical prediction needs to be verified by experimental observation.

4. Conclusions and discussion

Though a quasicrystal remains a brittle material in the range of low and normal temperatures, it may appear to have plastic flow at higher temperatures as pointed out in [1–7]. Meanwhile there may appear high stress concentration near a dislocation core or crack tip in the material. The stress level can go beyond the plastic limit τ_c (or σ_c) in the high stress zone. This also leads to plastic flow in quasicrystalline materials. References [1–7] predicted that the mechanism of plasticity reveals a dislocation motion. So a discussion of the interaction between a crack and dislocations is of interest. The results of (15) and (16) may be used as a parameter of the elasto-plastic fracture parameter for the material in the case of medium and large scale plastic deformations. The calculation in previous sections provides some quantitative pictures for the subject, where the classical BCS model for crystals is extended. If the phason parameters are absent, i.e., $w_i = 0, K_i = 0, R_i = 0, b_i^\perp = 0$, then solutions (13), (15) are reduced to the classical BCS solution for screw dislocations coupled with an anti-plane crack, where $A = C_{44}b/2\pi$, and b denotes the conventional Burgers vector. Solutions (13), (16) are then reduced to the classical BCS solution for edge dislocations coupled with a plane crack for hexagonal crystal where $A = (L+M)Mb/\pi(L+2M)$, $L = C_{12}$, $M = (C_{11} - C_{12})/2 = C_{66}$ and b represents the conventional Burgers vector. If the material is an isotropic one, i.e., $C_{12} = \lambda, C_{44} = C_{66} = \mu$ which are the Lamé constants, then the solutions reduce to the BCS solution for isotropic crystals, for which Bilby *et al* had given their solution for the case in [18, 19]. This confirms that our solutions are exactly correct physically and mathematically.

From our solutions it can be found that the influence of the phason field is evident. This influence lies in both the phason Burgers components $b_i^\perp$ and phason elastic constants K_i and phason–phonon coupling elastic constants R_i. For simplicity, in the calculations in a two-dimensional decagonal quasicrystal we need only consider the Burgers components $b_1^\parallel$ and $b_1^\perp$. The calculation is similar for other components.

Although the analysis is mainly concerned with the phonon field in the text, this is not in fact the case, as the phason and phonon fields are coupled, and they affect each other, as has been pointed out above. In addition an integral equation like (9) about the stress field H_{ij} and the solution can be obtained, and the derivation is similar. In that case the calculation is related to the generalized traction associated with the phason stress field, but the physical meaning for these parameters (the generalized traction) have not been made clear so far, so the calculation is not given here.

The solution of the interaction between a crack and a dislocation pile up for point group $8mm$ two-dimensional quasicrystals can also be derived in a similar manner. The results are not included however, as the formulation requires much space. The solution for point group $12mm$ two-dimensional quasicrystals is simpler, owing to decoupling between phonon and phason, so it is not included.

The singular integral equation method based on the known solution for a dislocation is simpler for determining some parameters of the interaction between a crack and a dislocation pile up in quasicrystals. But the method has its limitations. It cannot obtain the entire displacement and stress fields for the interaction problem. For this purpose one must use the complex variable function method as proposed in [10, 17]. Of course, with this method there are limitations too, which can solve those problems only, whose final governing equations must be harmonic, biharmonic and multiharmonic, otherwise it does not work.

In the previous discussion, the z axis was taken as the quasiperiodic direction of atomic arrangement for one-dimensional quasicrystals, and the periodic direction of atomic arrangement for two-dimensional quasicrystals; and it was further assumed that the cracks and dislocations are along this direction. If the cracks and dislocations are not along this direction, but along other directions, say along the x axis, the solutions will be changed because the boundary conditions for the cracks and dislocations are changed from those given by (7), (8) and (A.4), (A.8). The discussion will be lengthy and is avoided here due to limitations in space.

The interaction between cracks and dislocations for a large number of symmetries in quasicrystals, e.g. for icosahedral quasicrystals, is quite interesting. So far there is a lack of exact analytic solution of crack and dislocation interaction for icosahedral quasicrystals. We are presently carrying out such a study, and its results will be reported later.

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Appendix A. Fundamentals of stress analysis of dislocations in quasicrystals

For a one-dimensional hexagonal quasicrystal the screw dislocation problem is formulated by the equilibrium equations

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} = 0, \qquad \frac{\partial H_{zx}}{\partial x} + \frac{\partial H_{zy}}{\partial y} = 0, \tag{A.1}$$

the equations of deformation geometry

$$\begin{aligned} \varepsilon_{zx} &= \frac{1}{2}\frac{\partial u_z}{\partial x} = \varepsilon_{xz}, \qquad & \varepsilon_{yz} &= \frac{1}{2}\frac{\partial u_z}{\partial y} = \varepsilon_{yz} \\ w_{zx} &= \frac{\partial w_z}{\partial x}, & w_{zy} &= \frac{\partial w_z}{\partial y}, \end{aligned} \tag{A.2}$$

and the stress–strain relations

$$\begin{aligned} \sigma_{yz} &= \sigma_{zy} = 2C_{44}\varepsilon_{yz} + R_3 w_{zy} \\ \sigma_{zx} &= \sigma_{xz} = 2C_{44}\varepsilon_{zx} + R_3 w_{zx} \\ H_{zx} &= 2R_3\varepsilon_{zx} + K_2 w_{zx} \\ H_{zy} &= 2R_3\varepsilon_{yz} + K_2 w_{zy}. \end{aligned} \tag{A.3}$$

The final governing equations reduced from (A.1) to (A.3) are $\nabla^2 u_z = 0$, $\nabla^2 w_z = 0$, which are comparable to a fourth order partial differential equation of elliptic type, and the boundary conditions

$$\begin{aligned} &\sqrt{x^2 + y^2} \to \infty: \qquad \sigma_{ij} = 0, \quad H_{ij} = 0 \\ &u_z|_{y=0^+} - u_z|_{y=0^-} = b_3^{\parallel} \\ &w_z|_{y=0^+} - w_z|_{y=0^-} = b_3^{\perp}; \end{aligned} \tag{A.4}$$

the solution is given in the text.

For a point group $10mm$ two-dimensional quasicrystal, the straight 'edge' dislocation problem concerning Burgers components $b_1^{\parallel}$ and $b_1^{\perp}$ is formulated by the equilibrium equations

$$\begin{aligned} &\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0, \qquad \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \\ &\frac{\partial H_{xx}}{\partial x} + \frac{\partial H_{xy}}{\partial y} = 0, \qquad \frac{\partial H_{Yx}}{\partial x} + \frac{\partial H_{Yy}}{\partial y} = 0, \end{aligned} \tag{A.5}$$

the equations of deformation geometry

$$\begin{aligned} &\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \qquad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \qquad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) = \varepsilon_{yx} \\ &w_{xx} = \frac{\partial w_x}{\partial x}, \qquad w_{yy} = \frac{\partial w_y}{\partial y}, \qquad w_{xy} = \frac{\partial w_x}{\partial y}, \qquad w_{yx} = \frac{\partial w_y}{\partial x}, \end{aligned} \tag{A.6}$$

and the stress–strain relations

$$
\begin{aligned}
\sigma_{xx} &= L(\varepsilon_{xx}+\varepsilon_{yy}) + 2M\varepsilon_{xx} + R(w_{xx}+w_{yy}) \\
\sigma_{yy} &= L(\varepsilon_{xx}+\varepsilon_{yy}) + 2M\varepsilon_{yy} - R(w_{xx}+w_{yy}) \\
\sigma_{xy} &= \sigma_{yx} = 2M\varepsilon_{xy} + R(w_{yx}-w_{xy}) \\
H_{xx} &= K_1 w_{xx} + K_2 w_{yy} + R(\varepsilon_{xx}-\varepsilon_{yy}) \\
H_{yy} &= K_1 w_{yy} + K_2 w_{xx} + R(\varepsilon_{xx}-\varepsilon_{yy}) \\
H_{xy} &= K_1 w_{xy} - K_2 w_{yx} - 2R\varepsilon_{xy} \\
H_{yx} &= K_1 w_{yx} - K_2 w_{xy} + 2R\varepsilon_{xy}.
\end{aligned}
\tag{A.7}
$$

The final governing equation reduced from (A.5) to (A.7) is $\nabla^2\nabla^2\nabla^2\nabla^2 F = 0$, where F is a displacement potential function, so this is an eighth order partial differential equation of elliptic type, and the boundary conditions

$$
\begin{aligned}
&\sqrt{x^2+y^2} \to \infty: \qquad \sigma_{ij} = 0, \quad H_{ij} = 0 \\
&u_x|_{y=0^+} - u_x|_{y=0^-} = b_1^{\parallel} \\
&w_x|_{y=0^+} - w_x|_{y=0^-} = b_1^{\perp}
\end{aligned}
\quad . \tag{A.8}
$$

The solution is given in the text.

For a point group $10, \overline{10}$ two-dimensional quasicrystal, the formulation for the corresponding dislocation problem is similar to that for point group $10mm$ but instead of (A.7), we have

$$
\begin{aligned}
\sigma_{xx} &= L(\varepsilon_{xx}+\varepsilon_{yy}) + 2M\varepsilon_{xx} + R_1(w_{xx}+w_{yy}) + R_2(w_{xy}-w_{yx}) \\
\sigma_{yy} &= L(\varepsilon_{xx}+\varepsilon_{yy}) + 2M\varepsilon_{yy} - R_1(w_{xx}+w_{yy}) - R_2(w_{xy}-w_{yx}) \\
\sigma_{xy} &= \sigma_{yx} = 2M\varepsilon_{xy} + R_1(w_{yx}-w_{xy}) + R_2(w_{xy}+w_{yx}) \\
H_{xx} &= K_1 w_{xx} + K_2 w_{yy} + R_1(\varepsilon_{xx}-\varepsilon_{yy}) + 2R_2\varepsilon_{xy} \\
H_{xy} &= K_1 w_{xy} - K_2 w_{yx} - 2R_1\varepsilon_{xy} + R_2(\varepsilon_{xx}-\varepsilon_{yy}) \\
H_{yx} &= K_1 w_{yx} - K_2 w_{xy} + 2R_1\varepsilon_{xy} - R_2(\varepsilon_{xx}-\varepsilon_{yy}).
\end{aligned}
\tag{A.9}
$$

The final governing equation reduced from (A.5) to (A.7) is $\nabla^2\nabla^2\nabla^2\nabla^2 F' = 0$, which is similar to that for point group $10mm$ two-dimensional quasicrystals, but the definition of the displacement potential function F' is more complicated than that of F for point group $10mm$ and the solution is given in the text.

Appendix B. Physical meaning of the imaginary part of formula (12) being zero

From figure 1 it is seen that the fictitious crack (real crack plus the plastic zone or dislocation pile up) is subjected to an applied stress $-\tau^{(\infty)}$ (at $y = 0, |x| \leqslant l + d$) and the limit stress τ_s (at $y = 0, l < |x| < l + d$). For the applied stress $-\tau^{\infty}$, there is the stress intensity factor

$$K^{(1)} = \sqrt{\pi(l+d)}\tau^{(\infty)} \tag{B.1}$$

while for the limit stress τ_c, we have the stress intensity factor

$$K^{(2)} = -2\tau_c\sqrt{\frac{l+d}{\pi}}\cos^{-1}\left(\frac{l}{l+d}\right). \tag{B.2}$$

Because the material is in a plastic flow state, there is no stress singularity at the crack tip, so the total stress intensity factors must be zero, i.e.,

$$K^{\text{total}} = K^{(1)} + K^{(2)} = 0. \tag{B.3}$$

This leads to

$$2\tau_c \cos^{-1}\left(\frac{l}{l+d}\right) - \tau^{(\infty)}\pi = 0. \tag{B.4}$$

This is just (13) in the text, so the expression is the result of cancelling the stress singularity for the plastic flow state.

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